# An Overview of Research Highlights and Research-Related Activities

## I. Research on Graph Theory

### A. Graph Factors

Factors of graphs help you understand its structure, which is why it's been the subject of many researchers. A *factor* of a graph $G$ is a spanning subgraph of $G$ which is not totally disconnected. A factor is called a *degree-factor* if some degree condition is imposed for each vertex of the factor. Among many early results on degree-factors, the 1-Factor Theorem by W. Tutte [A-1] and the $[f,g]$-factor Theorem by L. Lovasz [A-2] are critical.

Akiyama and M. Kano [A-3] introduced a concept called *component-factor* of $G$, in which each component of the factor is isomorphic to some graphs such as a path, a tree, a star, etc. The introduction of the concept of component-factors expanded the research on factors and factorization theory. Akiyama, Era, and Avis obtained the Path-Factor Theorem [A-4] using alternating paths. Akiyama and Kano [A-5] also obtained a necessary or sufficient condition for the existence of a $P_3$-factor and $P_4$-factor. As an application of the $P_3$-factor, Akiyama and Kano [A-6] considered the problem of packing triominos into a truncated chessboard. Generalizing the latter, V. Chvátal [A-7] studied the problem of packing paths $P_3$ into a given graph from an algorithmic point of view.

A Hamilton cycle is a connected 2-fator. It was shown by Akiyama, M. Kobayashi and G Nakamura in [A-8] that there is a symmetric Hamilton cycles decomposition of $K_n$ that is not isomorphic to the Walecki decomposition $W_n$, for all odd $n > 7$.

In 1985, Akiyama and Kano published a review article of the previous work on factors and factorizations of graphs in the Journal of Graph Theory [A-9]. In 2007, they also published a book (under Springer [A-10]) titled "*Factors and Factorizations of Graphs: Proof Techniques in Factor Theory*," which comprehensively summarizes the theory of factors and graph factorizations and organizes and explains the methods used to prove theorems on graph factors. This book is now cited in almost all recent papers on graph factors.

### B. $G, \bar{G}$ Series (Both $G, \bar{G}$ satisfies a condition $P$)

Let $P$ be a prescribed condition for any graph. In 1978, Akiyama and Harary [B-6] introduced a series of problems aimed to determine all graphs $G$ such that both $G$ and $\bar{G}$ (the complement of $G$) satisfy the given ondition $P$. Various results were obtained for the following $P$: $n$-connectedness [B-1], equal girth or circumference [B-2], self-complementary block [B-3], a specified number $k$ of endvertices [B-4], equal chromatic numbers [B-5], contraction critically $k$-connected [B-7], interval graphs [B-8], equi-eccentric graphs [B-9], etc. Research on $G, \bar{G}$ satisfying a common property $P$ has some interesting applications. For example, a graph $G$ may represent a network of roads in a city, a distribution network, a sewerage system, or a nerve or vascular network in the human body. In this case, when a network $G$ fails with respect to the condition (function) $P$, then $\bar{G}$ can be used in place of $G$ (assuming $\bar{G}$ satisfies $P$).

### C. Linear Arboricity

The linear arboricity of a graph is closely related to the design of a database retrieval system. A graph $G$ is a *linear forest* in which each component is a path. Supposing that the edge set of $G$ is partitioned into subsets such that each one is a linear forest. Then the minimum number of subsets is defined as the *linear arboricity* of $G$, denoted by $\ell_a(G)$. In 1976, Akiyama conjectured that the linear arboricity for $r$-regular graphs is $\ell_a(G) = \lceil (r+1)/2 \rceil$. This was proven to be true for cases $r = 3, 4$ in his papers with $G$. Exoo and F. Harary [C-1, C-2, C-3, C-4]. Unfortunately, nearly half a century later, this conjecture remains unresolved for many cases, although many researchers have validated this conjecture for relatively small values of *r*.

### D. Path Chromatic Numbers and Conjecture on Maximum Induced Forests

The four-color problem for map coloring has been a well-known driving force in the development of graph theory. In 1977, the proof of K. Appel, W. Haken, and J. Koch [D-1], with the help of computers, finally put an end to this difficult problem. However, their paper was more than a hundred pages long, and many graph theorists are still yearning for a more graph-theoretic approach.

Akiyama also joined in this search to find a simpler proof of the four-color theorem without the help of computers, which led him to introduce the generalized chromatic numbers (called *path chromatic numbers*). Akiyama, H. Era, M. Watanabe, and S. Gervacio [D-2] defined the path chromatic number as an extended concept of the chromatic number as follows: The *k-path chromatic number* $\chi(G;P_k)$ of a graph $G$ is the minimum number $n$ such that the point set of $G$ is partitioned into $n$ subsets, where the subgraph induced by each subset is a union of paths of order at most $k$, denoted by $\chi(G;P_k)$. Then, $\chi(G;P_1)=\chi(G), \chi(G;P_1) \geqq \chi(G;P_2) \geqq \cdots \geqq \chi(G;P_\infty)$ holds. In this paper, it is shown that $\chi(G;P_\infty) \leqq 2$ for any outerplanar graph, and that a sequence of planar graphs $G$, such that $\chi(G;P_k)=4$ for any integer $k$, can be constructed.

Another parameter that can be generalized is the point-independent number of graph $G$, denoted by $\beta_o(G)$. Based on the 4-color theorem, $\beta_o(G) \geqq \lceil \frac{p}{4} \rceil$ for a planar graph $G$ of order $p$. Extending the notion of a point - independent number, Akiyama and Watanabe defined the *point-independence forest number* $\beta_F(G)$ as the maximum cardinality of a subset of points in a graph $G$ such that the subgraph of $G$ induced by that subset is a forest. Akiyama and Watanabe conjectured that $\beta_F(G) \geqq \lceil \frac{p}{2} \rceil$ if $G$ is a planar graph of order $p$, and showed examples of graphs that achieve the lower bound for each $p$. Similarly, they conjectured that $\beta_F(G) \geqq \lceil \frac{5p}{8} \rceil$ if $G$ is a planar bipartite graph of order $p$, and showed examples of a sequence of graphs that achieve the lower bound for each $p$ [D-3]. P. Erdös finds their conjecture interesting because, if it were proven to be true, then one can show that $\beta_o(G) \geqq \lceil \frac{p}{4} \rceil$ for a planar graph $G$ of order $p$, without using the 4-color theorem.

## E. Middle Graphs, Eccentric Graphs: Graph Equations

Akiyama, T. Hamada and I. Yoshimura introduced the concept of *middle graphs* and showed their characterization. They studied the connectivity, the edge-connectivity, and arboricity [E-1, 2, 3] of a middle graph.

The *eccentric graph* $G_e$ of $G$ is defined on the same set of vertices of $G$ and joining two vertices in $G_e$ with an edge if and only if one of the vertices has maximum possible distance from the other in $G$. Akiyama, K. Ando and D. Avis studied miscellaneous properties of the eccentric graphs, and characterized graphs with $G_e = K_p$, $G_e = pK_2$ and $G_e = \bar{G}$ [E-4, 5, 6]. Akiyama, T. Kodate and K. Matsunaga characterized graphs which are clusters in [E-7].

In addition, Akiyama studied many graph equations together with S. Simic, Hamada, and others, which has led to numerous results [E-8-10]. In particular, they determined exactly 23 $(G,H)$ solutions for the graph equation $\overline{L(G)} = L(H)$[E-11].

## F. Research-Related Activities

Until the early 1970s, no research ~~results~~ on graph theory was ever presented in the Mathematical Society of Japan. In 1974, Akiyama presented his research on graph theory for the first time at the Mathematical Society of Japan. Since then, and in almost every succeeding year, he was able to present his results on graph theory at the Mathematical Society of Japan and at the Research Institute for Mathematical Analysis (in Kyoto University). In 1982 and 1990, he gave two special lectures on graph theory at the Applied Math Division of the Mathematical Society of Japan. Since then, the number of researchers in graph theory has gradually increased and the quality and quantity of research results have improved [F-1, F-2]. To this day, Akiyama continues to receive many invitations as plenary or keynote speaker in various international conferences on graph theory and combinatorics.

Akiyama was the first person in Japan to be awarded a doctorate of science specializing in the area of graph theory. In 1979, he became the first Asian to serve as editor of the Journal of Graph Theory (published by Wiley), where he edited and reviewed many papers on graph theory for 18 years (until 1997). In 1986 and 1990, Akiyama organized the International Conference on Graph Theory in Japan, gathering about 300 participants including leading researchers like S. Hitotumatu. P. Erdös, R. Graham, B. Bollobas, V. Chvátal, N. Alon, V. Sos, E. Szemerédi, J. Urrutia, J. Harary, M. Deza, J. Pach, and L. Lovász [F-3, 4]. The proceedings of the said conference were published in a special issue of Discrete Math and Annals of Discrete Math [F-5]. Also in 1986, Akiyama launched Graphs and Combinatorics – a journal of graph theory and combinatorics published by Springer [F-6]. The journal has now grown into a high impact factor journal which publishes many high quality research papers.

## II. Research on Discrete and Computational Geometry

### G. Existence on Disjoint Simplices and Noncrossing Edges of d-Dimensional Geometric Hypergraphs

In 1985, Akiyama and N. Alon [G-1], showed the following theorem using the Ham-Sandwich theorem obtained from the Borsuk-Ulam theorem: Let $A$ be a set of $d \cdot n$ points in $R^d$ and let $A = A_1 \cup A_2 \cup \cdots \cup A_d, |A_i| = n (1 \leqq i \leqq d)$ be a direct sum partition of $A$, then there exist $n$ disjoint $(d-1)$-dimensional simplices, each containing precisely one vertex from each $A_i (1 \leqq i \leqq d)$. Using $P$. Erdös' result [G-2], the aforementioned theorem can be extended as follows: Any $d$-dimensional geometric hypergraph with $n^{d-\left(\frac{1}{\ell^{d-1}}\right)}$ edges has $\ell$ edges that do not cross each other. This seems to be of interest to researchers, as evidenced by its number of citations, because it uses the Ham-Sandwich theorem for discrete objects, yet is shown to be applicable to $d$-dimensional geometric hypergraphs.

### H. Alternating Paths

Assume that there are $2n$ points in the plane in general position colored with red or blue so that the number of red and blue points is the same. In this case, Akiyama and J. Urrutia [H-1] showed an algorithm in $O(n^2)$-time to determine if there exists an alternating path connecting red and blue points which does not intersect. They also presented a specific procedure for constructing such a non-intersecting alternate path, which is useful in many matching problems.

### I. Balanced Coloring for Lattice Points

Consider a set $P_n$ consisting of $n$ lattice points in the 2-dimensional coordinate plane. The $m$-*coloring* of $P_n$ is the partition $P_n = S_1 \cup S_2 \cup \cdots \cup S_m$. A *balanced m-coloring* of $P_n$ means that for any line $L$ parallel to the coordinate axes, the number of elements in $S_\ell$ on $L$ and the number of elements in $S_k$ on $L$ are either equal or differ by one, for any $\ell, k (1 \leqq \ell, k \leqq m)$. Akiyama and J. Urrutia [I-1] showed, using König's *1-factorization* theorem for regular bipartite graphs, that for any $m$ $(2 \leqq m \leqq n)$, there exists a balanced $m$-color of $P_n$. This result was extended to the problem of balanced coloring for higher-dimensional lattice point sets and was shown to be applicable to block designs.

### J. Purely Recursive k-dissections of Polygons

The problem of dissecting polygons has a long history [J-1]. Given a polygon $P$, we say that D is a $k$-*dissection* of $P$ if it is a dissection of $P$ into $n$ pieces $\{P_1, P_2, \cdots, P_n\}$, which can be rearranged to form $k$ polygons similar to $P$ of different sizes. $D$ is a *purely recursive* $k$-*dissection* of $P$ if $D$ is a $k$-dissection and is a cut such that the pieces of $P$ can be rearranged to form 2, 3, $\cdots, k$ successive polygons similar to $P$. Akiyama, T. Sakai, and J. Urrutia [J-2] showed the existence of recursive k-dissections for arbitrary triangles, quadrilaterals, pentagons, hexagons, and other arbitrary regular polygons. Moreover, if $P$ is a square, Akiyama, Nakamura, A. Nozaki, K. Ogawa and Sakai [J-3, 4] constructed a purely recursive $n$-dissection of $P$ with $2n + 5$ pieces, and proved that this dissection has the smallest number of pieces.

### K. Distances between Vertices of a Polyhedron and a Polytope

Let $P$ be a polyhedron with $v$ vertices, labeled $P_1, P_2, \cdots, P_v$. Akiyama and I. Sato [K-1] proved that $\sum |P_i P_j|^2 = v^2$ for any regular $n$-dimensional polytope $(n \geqq 2)$ with $v$ vertices $P_1, P_2, \cdots, P_v$ inscribed in a unit $n$-sphere, where the summations is taken under $1 \leqq i < j \leqq v$. Note that this result is dependent only on the number of vertices of $P$ and not its dimension. Furthermore, they showed in [K-2] the upper and lower bounds antipodal distance of ~~a~~ Wythoffian polytopes.

### L. Research-Related Activities (JCDCG)

In 1997, Akiyama organized the first Japan Conference on Discrete and Computational Geometry (*JCDCG*) in Tokyo. Since then, the JCDCG has been held annually around Asia except in 2008 and 2020. The 25th JCDCG conference was held in Bali, Indonesia last in 2023. Akiyama has served as the conference chair (or conference co-chair) from the first to the 25th conference. The papers presented at each conference have been carefully reviewed and sixteen issues of proceedings have been published by *Lecture Notes in Computer Science, Computational Geometry: Theory and Applications, Graphs and Combinatorics, Thai J. Math and J. Information Processing,* among others. JCDCG is now regarded as a reputable international conference, with regular attendees from CCCG in Canada and ECG in Europe.

## III. Polygons, Polyhedra, and Polytopes

### M. Tile-Maker

Akiyama has studied various properties of Conway tiles [M-1, 2]. A polyhedron (including a dihedron) $P$ is a *tile-maker* if any net $N$ of $P$ is a tile. That is, the plane can be tessellated with copies of $N$. Akiyama [M-2, 3] determined all polyhedra (including dihedra) that are tile-makers using the Conway Criterion. The simplicity of this result has since gained the interest of mathematics educators and students. Akiyama also examined which among the 17 types of periodic plane tiling could be constructed [M-2]. S. Langerman and A. Winslow [M-4] later on extended these results to determine all tile-makers for flat closed-surfaces such as torus, klein bottle, and projective plane using the Gauss-Bonnet theorem.

### N. Tessellation Polyhedra

In relation to the tile-maker problem, Akiyama, etc. began investigating all tessellating polyhedra. Among the infinitely many convex polyhedra, there are some whose faces are all regular polygons, or *RFP,* short for "*regular-faced polyhedra*."

A polyhedron $P$ is called a *tessellation polyhedron* if at least one of its $e$-nets (nets obtained by cutting $P$ along its edges) tiles the plane. Akiyama, S. Langerman. G. C. Shephard etc. [N-1] proved that there are exactly 23 tessellation polyhedra with RFPs. Since there are astronomically many different $e$-nets of each RFP, for example, the Johnson polyhedron $J_{44}$ has 5, 295, 528, 588 different e-nets, the team relied on a computer to check which $e$-nets are tiles, making use of the Conway criterion.

### O. Reversibility (Hinged Rotational Equi-Decomposability)

Polygons $P$ and $Q$ are said to be *equi-decomposable* if $P$ can be cut into a finite number of pieces, which can be rearranged to form $Q$. $P$ and $Q$ are equi-decomposable if and only if P and Q have the same area, a known result that was proven independently by three mathematicians during the late 19th century, namely, F. Bolyai [O-1], P. Gerwien [O-2], and W. Wallace [O-3] using a constructive approach. D. Hilbert's well-known problem on equi-decomposability of a tetrahedra of the same base and same height was solved by M. Dehn [O-4] by defining Dehn invariants for polyhedra and showing counterexamples.

In 1907, H. Dudeney proposed the following problem: If $P$ and Q have the same area, where $P$ is an equilateral triangle and $Q$ is a square, is it possible to dissect $P$ into several pieces and rearrange them to form Q? [O-5]. Dudeney's technique is to cut an equilateral triangle into four pieces, attach a "hinge" to the vertex of each piece, connect the pieces in a chain, and then rearrange~~d~~ the pieces to form a square. In this case, the perimeter and interior of the equilateral triangle and square are interchanged (often called the *inside-out property*). Akiyama and Nakamura [O-6] extended this concept and defined a pair of polygons $P$ and $Q$ to be *reversible* if there is a hinged transformation between $P$ and $Q$ satisfying the inside-out property. They specifically determined reversible pairs for arbitrary triangles, squares, and pentagons, apart from equilateral triangles and squares [O-7, 9, 10, 11, 12, 13]. Akiyama and Matsunaga extended the notion of transformation not only to polygons but also to pairs of arbitrary figures surrounded by curves [O-8]. Not only that, Akiyama with H. Seong also showed an algorithm for determining whether a given polygonal pair is reversible or not [O-14, 15]. Akiyama, Matsunaga, and S. Langerman [O-16] showed that a pair of non-overlapping nets of any polyhedron is reversible. The converse of this theorem is also true as shown in another paper by Akiyama, E. Demaine, and S. Langerman [O-17, M-2]. In other words, these results prove that a reversible pair is, in fact, a non-overlapping net of some common polyhedra. This puts an end to the theory of reversibility, originally developed from the Haberdasher's Puzzle.

### P. Minimum Perimeter Nets for the Platonic Solids

Determining a net with the minimum perimeter length (MPL) for a given polyhedron is reduced to a

problem of finding the minimum Steiner tree spanning all vertices of the polyhedron. Akiyama, X. Chin, and M. J. Ruiz [P-1] found the nets with minimum perimeter lengths (NMPL) of Platonic solids by using Melzak's Algorithm for solving the minimum Steiner tree problem. The minimum perimeter lengths of unit Platonic solids are given below:
MPL of a unit regular tetrahedron is $2\sqrt{7}$ (≒ 5.29150),
MPL of a unit cube is $2(2\sqrt{3}+3)$ (≒ 12.92820)
MPL of unit regular octahedron is $2\sqrt{19}$ (≒ 8.71780)
MPL of unit regular dodecahedron is 37.19729 and
MPL of unit regular icosahedron is $2(\sqrt{37}+2\sqrt{3})$ (≒ 19.09370).

## Q. Element Numbers of Polyhedral Families

H. Minkowski [Q-1] showed that there are only five types of parallelohedra (called also Fedorov's polyhedra) that can fill space only by parallel translations (assuming that those congruent by affine transformation are in the same family). Akiyama defined the number of elements for a set of polyhedra as follows: Let $\Sigma, \Omega$ be a set of polyhedra. If any element of $\Sigma$ can be composed of a finite number of elements of $\Omega$, then $\Omega$ is called an *element set* of $\Sigma$. That is, $\forall P \in \Sigma,\ P = \bigcup_{i\in\mathbb{Z}} n_i\sigma_i$ ($\mathbb{Z}$ is the finite set of integers, $0 \leqq n_i \in \mathbb{Z}$, $\sigma_i \in \Omega$.) The *element number* is the smallest number of the size of the element set $\Omega$ of $\Sigma$. Akiyama, with Ikuro Sato and others, determined the element number for several important families of polyhedra, such as the family of parallelohedra, the family of regular polyhedra, and the family of regular polytopes, using Dehn invariants and other methods [Q-2, 3, 4, 5]. More importantly, they secured a patent for the *pentadron*, the atom (element) of a parallelohedra whose element number is 1. In addition, the element number for the family of 92 Johnson Zalgaller polyhedra is conjectured to be 33.

## R. Reversibility of Polyhedra（Hinged-Transformation with Inside-out Properties）

Akiyama extended the concept of reversibility for two-dimensional figures to three-dimensional polyhedra. By considering the Voronoi domains of face-centered cubic (FCC), body-centered cubic (BCC), and hexagonal closest packing (HCP) lattices, Akiyama, Sato, et al. [R-1, 2] showed that any two parallelohedra can be reversible. Since this study is closely related to the crystal structure and the mechanism of phase transformation, it also got the attention of those studying chemistry and crystallography [R-3].

## S. Universal Measuring Box

It has been known since the Edo period that by using a box with volume $6\ell$, one can accurately pump out $1\ell, 2\ell, \cdots, 6\ell$ of water. An unmarked box that can pump out $1\ell, 2\ell, \cdots, n\ell$ of water after dipping it into a water barrel exactly once is called a *universal measuring n-box* (or simply an $n$-*box*). Various shapes of universal measuring boxes have been studied in [S-1]. An $n$-box with base B is called *orthogonal type* if each face other than B is orthogonal to its base B. Akiyama, H. Fukuda, C. Nara, T. Sakai and J. Urrutia. [S-2] found two orthogonal 41-boxes with triangular bases, and a non-orthogonal 127-box with a triangular base. Among all $n$-boxes of this type, $n = 127$ is the maximum. It was also proved in [S-1] that there exist orthogonal 858-boxes with quadrangular bases.

# IV. Game, Knot and Puzzle

## T. Nim-like Games

Akiyama and G. Nakamura investigated in finding winning strategies for many recreational games like sliding Puzzle, Peg Solitaire Nim Game, etc. and published a book [U-1] from Morikita Publishing Co. in 1998. T. Ooya and Akiyama introduced in [T-2] a few games in which binary and Fibonacci expansions of numbers play a prominent role for binding winning strategies.

## U. Möbius Flowers

Bisecting conjoined Möbius bands along each centerline, some of them end up as interlocking hearts, while the rest end up as two separated hearts. What indicators allow us to differentiate between the two cases?

Akiyama etc. revealed in [K-2, U-1] math behind and generalized the number of conjoined Möbius bands to any natural number greater than 2.

## V. Dudeney's Round Table Problem

In 1905, Dudeney [V-1] proposed the Round Table Problem as follows: "Seat the same $n$ persons at a round table on $(n-1)(n-2)/2$ occasions so that no person shall ever have the same two neighbours twice. This is equivalent to saying that every person must sit once, and only once, between every possible pair."

The problem is equivalent to asking for a set of Hamilton cycles in the complete graph $K_n$ with the property that every path of length two lies on exactly one of the cycles. M. Kobayashi, J. Akiyama and G. Nakamura [W-2] gave a solution for the problem when $n = p + 2$, where $p$ is an odd prime number such that 2 is the square of a primitive root of $GF(p)$, and $p \equiv 3\ (mod\ 4)$.

## W. Other Research-Related Works

In 1995, Akiyama planned the publication (via Asakura Publishing) of a series of lectures on finite discrete mathematics. He co-authored the first volume with R. L. Graham [W-1] titled "Introduction to Discrete Mathematics" and wrote the second volume titled "Frontiers of Graph Theory" [W-2]. In 2020, Akiyama published "Frontiers of Discrete Geometry" with Kindai Kagakusha (Modern Science) [W-3].

In 2007, the Akiyama-Chvátal Sixtieth Birthday Commemorative International Conference took place at Kyoto University Clock Tower Hall. A book titled "Computational Geometry and Graph Theory, —The Akiyama-Chvátal Festschrift," edited by D. Avis, A. Bondy, M. Kano, and N. Katoh was published by Springer [L-9].

In 2015, Akiyama and Matsunaga published "Treks into Intuitive Geometry — The World of Polygons and Polyhedra" in Springer [M-2]. In less than a decade since its release, many significant results in this field were obtained, and so an updated second edition is published in 2024.

As of November 2023, Akiyama has authored 182 papers with more than 1600 citations, which have been read by at least 36,000 people worldwide (source: Research Gate).

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